Since the average treatment effect (ATE) is just the contrast $\mathbb{E}[Y(1)] - \mathbb{E}[Y(0)]$

as

As an extension of [Athey et al., 2018], Bradic et al. [2019a

the same length, **v**

mension $\mathbf{p}$ grows with $\mathbf{n}$ and $\mathbf{n} \equiv \mathbf{n}_{\text{aux}}$

where $\boldsymbol{\varepsilon_i}$

For the sake of completeness, a corresponding lower bound proof can be found in Appendix

$\mathbf{b}$ respectively. Similarly, we denote $\boldsymbol{\theta}$

Jana Janková and Sara van de Geer. Semiparametric efficienc

Ya’acov Ritov, Peter J Bickel, Anthony C Gamst, and Bastiaan JanJandn

immediate that the matrix $\tilde{\mathbf{X}}$ is almost surely full row rank, so that there is at least one $\mathbf{y}$ such that almost surely the following equality holds,

$$\tilde{\mathbf{X}}\mathbf{y} = \boldsymbol{\varphi}$$

As explained in Remark 2, the last term in the above decomposition has mean zero by design of the program (8). Invoking constraint (9) and Condition 7

where for the last inequality we decompose $\tilde{\mathbf{x}}$

A.6 Proof of Theorem 6

A.6.1 Proof of Theorem 6(i)

Using the same analysis as in Lemma 1

Lemma 14. *Under the conditions of Theorem 6(ii), we have that*

$$\left|\tilde{\sigma}_e^2 - \bar{\sigma}_e^2\right| = o_P(1/\sqrt{n}),y$$

Next, we compute the χ^2

For **(II)**